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Plenary Lectures

MODELING OF FUNCTIONALLY GRADED SMART PLATES WITH GEOMETRIC NONLINEARITY AND GRADIENT ELASTICITY EFFECTS

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Summary: In this paper, equations of motion of plates with surface-mounted piezoelectric layers, while accounting for the gradient elasticity through modified couple stress model, the von Kármán nonlinearity, and linear piezoelectricity are developed using Hamilton's principle. The formulation includes the coupling between mechanical deformations and the charge equations of electrostatics. The mathematical model developed herein is hybrid in the sense that an equivalent single layer theory is used for the mechanical displacement field whereas the potential function for piezoelectric laminae are modeled using a layerwise discretization in the thickness direction. For the equivalent single layer, the Reddy third-order shear deformation theory is used. The approach described here is that standard plate models can be enhanced to include the coupling between the charge equations and the mechanical deformations as well as the modified couple stress effect.

Extended Abstract

In this paper, Hamilton's principle is used to derive a set of governing equations of a plate with surface-mounted piezoelectric layers. The charge equations of electrostatics are coupled to the mechanical deformations by using a modified energy density function given by [1]

$$L = \int_{\Omega} \left[\tfrac{1}{2}\rho \dot{u}_i \dot{u}_i - H\left(\varepsilon_{ij}, \chi_{ij}, E_i\right)\right] dx\, dy\, dz$$

where $H\left(\varepsilon_{ij}, \chi_{ij}, E_i\right)$ is called the electric enthalpy density function having the strain tensor ε_{ij}, the components of the symmetric curvature tensor χ_{ij}, and electric field E_i as arguments, ρ is the mass density, and $\dot{u}_i$ is the time derivative of the *i*-th displacement component. Numerous field theories can be derived based upon the particular selection of $H\left(\varepsilon_{ij}, E_i\right)$. Of course, they must all follow according to the conservation laws of linear and angular momenta as well as energy. For this paper $H\left(\varepsilon_{ij}, E_i\right)$ is taken as

$$H(\varepsilon_{ij}, E_i) = \frac{1}{2} C_{ijkl}\varepsilon_{ij}\varepsilon_{kl} - \mu l^2 \chi_{ij}\chi_{ij} - e_{ijk}E_i\varepsilon_{jk} - \frac{1}{2} k_{ij}E_iE_j$$

where C_{ijkl}, e_{ijk} and k_{ij} are called the elastic, piezoelectric stress and dielectric permittivity constants, respectively, and μ is the shear modulus, l is the material length scale, and χ_{ij} are the components of the symmetric curvature tensor [2]:

$$\mathrm{X} = \frac{1}{2}\left[\nabla\omega + (\nabla\omega)^{\mathrm{T}}\right], \quad \omega = \frac{1}{2}\nabla \mathrm{x}\, \mathbf{u}$$

The linear theory of piezoelectricity is based upon an electrodynamic approximation which allows for the electric field E to be derivable from a scalar potential function ϕ as follows:

$$\mathrm{E} = -\nabla\phi\left(E_i = -\frac{\partial\phi}{\partial x_i}\right)$$

The Reddy third order shear deformation [2] is based on the displacement field

$$u_1(x,y,z) = u(x,y) + f(z)\,\psi_x(x,y) + c_1 z^3\left(-\frac{\partial w}{\partial x}\right)$$

$$u_2(x,y,z) = u(x,y) + f(z)\,\psi_y(x,y) + c_1 z^3\left(-\frac{\partial w}{\partial y}\right)$$

$$u_3(x,y,z) = u(x,y)$$

where $f(z) = z - c_1 z^3$ and $c_1 = (4/3h^2)$, h being the plate thickness.

A complete theory of plates with surface mounted piezoelectric layers is developed with the help of Hamilton's principle

$$0 = \delta\int_0^T L\, dt$$

The dependent variables of the theory are the five generalized displacements from the kinematics and the electric potential from the piezoelectric layers.

References

1. J. A. Mitchell and J. N. Reddy, "A Refined Hybrid Plate Theory for Composite Laminates with Piezoelectric Laminae," International Journal of Solids and Structures, 32(16), 2345-2367, 1995.
2. J. N. Reddy and J. Kim, "A nonlinear modified couple stress-based third-order theory of functionally graded plates," Composite Structures, 94, 1128-1143, 2012.
3. J. N. Reddy, Reddy, "A simple higher-order theory for laminated composite plates," Journal of Applied of Mechanics, 51,745-752, 1984.